

INJURY BIOMECHANICS RESEARCH
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ANALYSIS OF INJURY VS. EXPOSURE DATA USING WEIBULL DISTRIBUTION
 FUNCTION

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INTRODUCTION

The author and his colleagues have analyzed biomechanical data -- especially cadaver impacts -- for a number of years. As other researchers would critique the approach taken in that work, the methods of analysis broadened and changed over the years. Some changes were due to a reconsideration of the validity of the mathematical method which was being employed. For example, regression analysis is designed for a dependent variable which is continuous and has a nominally linear relationship with the independent variables; but the AIS (a measure of injury frequently employed in biomechanical analyses) is neither a continuous variable nor a linear descriptor of injury outcome.

Other changes in the approach to analysis were due to a deeper reflection upon the way in which the biomechanical experiments are conducted. In most biomechanical experiments, the specimen is tested in a "one shot" fashion at one particular velocity or force level but not at progressively increasing values from zero up to the measured velocity or force. Biomechanical experiments are not like "light bulb experiments" where one starts with a thousand light bulbs turned on at day one and leaves the bulbs on continuously in order to record the time at which each bulb burns out. In other words, the independent variables (viz, force measured in dynamic experiments on bone shafts or velocity measured in pendulum impacts to the thorax) are not "excited" continuously from "zero" to the point where the value of the independent variable is recorded.

A recent paper [1]* which discussed the manner in which biomechanical experiments are conducted is by Ran, Koch, and Mellander of the Volvo Car Corporation in Goteborg, Sweden. This paper pointed out that statisticians refer to data which has a known bias as censored data. Censored data are frequently treated by statisticians using the Maximum Likelihood Method. The Maximum Likelihood Method and the Weibull Distribution are used in the Volvo paper to analyze several typical biomechanical problems.

Having read the Volvo paper on the Maximum Likelihood Method and the Weibull Distribution, the author decided to apply the method suggested by Volvo to familiar biomechanical problems previously solved by other methods. The purpose of this paper is to apply the method used by Volvo to additional data sets and share an assessment of the success of the approach.

REVIEW OF APPROACH

The Volvo paper uses the Weibull Cumulative Distribution Function and interprets it as the probability of a response (such as the occurrence of an injury of severity AIS ≥ 3) given a specific value of

* Numbers in brackets designate references at end of paper.

the independent explanatory variable. The Weibull Cumulative Distribution Function has the form

$$W(z; \alpha; \beta; \gamma) = 1 - e^{-[(z - \gamma)/\alpha]^\beta}$$

where z is the independent variable (which is the laboratory measured or processed variable and has a bias because of the manner in which the experiment is set-up, e.g., the experimental velocity is forced to fall in the band and or bands of interest) and α , β , and γ are parameters to be determined. The Weibull Function might be interpreted as the probability of bone fracture as shown in Figure 1. The symbol W denotes the particular cumulative distribution function which is the Weibull Function and the symbol F denotes a general cumulation distribution function for which no assumption has been made about the underlying population. Reference 2 is a good Weibull Function introductory text with an engineering point of view.

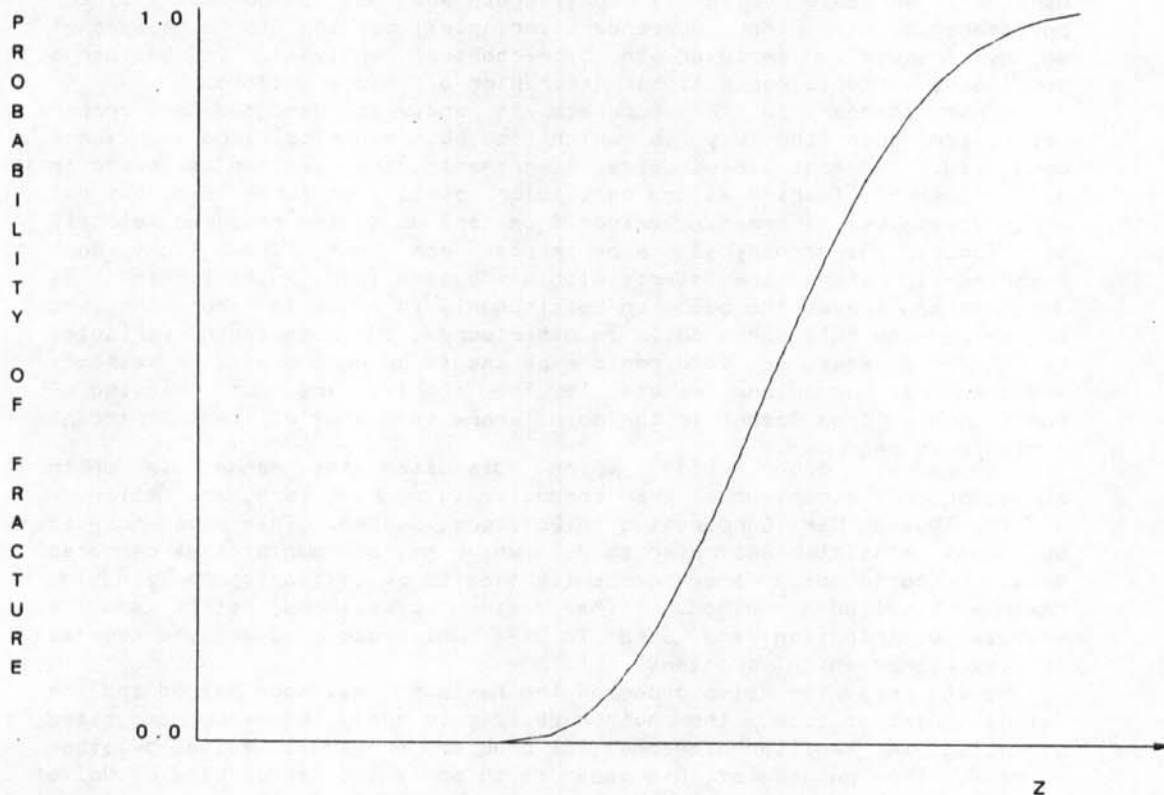


Figure 1 - Weibull Function is Interpreted as Probability of Response (Fracture).

The method suggested by Volvo begins with a plot of all-or-nothing -- or quantal -- responses versus the biased independent variable as shown in Figure 2. In this example there are four data points -- two non-fractures and two fractures -- corresponding to the biased independent variables

Z_1, Z_2, Z_3 , and Z_4 . Since the assumed probability of a fracture at Z_1 is $W(Z_1)$ -- where $W(Z)$ is a shortened notation for the Weibull function $W(Z; \alpha, \beta, \gamma)$ -- then the probability of a non-fracture is $1 - W(Z)$. It follows that the probability of the occurrence of the four data points in Figure 2 is :

$$L = [1 - W(Z_1)][1 - W(Z_2)][W(Z_3)][W(Z_4)]$$

where L is referred to as the likelihood function. The α, β , and γ are then found which maximize L . The maximized L is called the Maximum Likelihood Function. In other words, the method suggested by Volvo chooses from among all the possible values of α, β , and γ those three values that maximize the probability of obtaining the sample that was obtained.

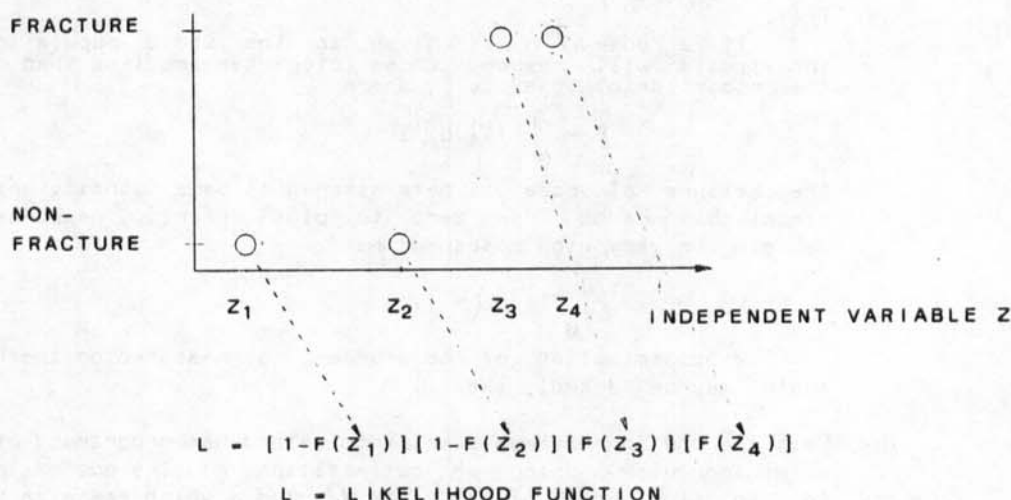


Figure 2 - Explanation of Approach used in Reference 1.

The probability of a fracture is then a plot of $W(Z, \alpha, \beta, \gamma)$ -- where α, β , and γ are those which maximized L -- versus Z .

The method is statistical in nature. One of the more concise statements of the connection between quantal data and the statistical approach is by Finney [3]:

" When the characteristic response is quantal, its occurrence or non-occurrence will depend upon the intensity of the stimulus applied. For any one subject, under controlled conditions, there will be a certain level of intensity below which the response does not occur and above which the response does occur; in psychology such a value is designated the threshold ..., but in pharmacology and toxicology the term tolerance seems more appropriate. This tolerance value will vary from one member to another of the population used, frequently between quite wide

limits. When the characteristic response is quantitative, the stimulus intensity needed to produce a response of any given magnitude will show similar variation between individuals. In either case, the value for an individual is likely to vary from one occasion to another as a result of uncontrolled internal or external conditions....

For quantal response data it is therefore necessary to consider the distribution of tolerances over the population studied. If the dose, or intensity of the stimulus, is measured by λ , the distribution of tolerances may be expressed by

$$dF = f(\lambda)d\lambda;$$

this equation states that a proportion, dF , of the whole population consists of individuals whose tolerances lie between λ and $\lambda + d\lambda$, where $d\lambda$ represents a small interval on the dose scale, and that dF is the length of this interval multiplied by the appropriate value of the distribution [density] function, $f(\lambda)$.

If a dose λ_0 is given to the whole population, all individuals will respond whose tolerances are less than λ_0 , and the proportion of these is F , where

$$F = \int_0^{\lambda_0} f(\lambda)d\lambda;$$

the measure of dose is here assumed to be a quantity which can conceivably range from zero to plus infinity, response being certain for very high doses so that

$$\int_0^{\infty} f(\lambda)d\lambda = 1.$$

The distribution of tolerances, as measured on the natural scale, may be markedly skew...."

The Weibull Function/Maximum Likelihood method was programmed on a VAX 11/780 (a minicomputer) using an optimization routine out of the IMSL library [4] to find the values of α , β , and γ which maximize L . The program was then validated against known solutions in the literature and in particular against the recorded values of α , β , and γ in Reference 1.

METHOD APPLIED TO SIDE IMPACT ANALYSIS

Having the Maximum Likelihood Method and Weibull Distribution method up and running on the computer, the first step was to revisit a previous analysis in the side impact area [5]. Basically, Reference 5 concerns the investigation of a functional relationship between the intensity of the mechanical input to 61 cadavers and the injury as measured by the AIS scale. The signals from 12 of the cadaver tests did not pass quality control such that only 49 were used in the actual analysis. Reference 5 analyzed these 49 cadaver tests using the "Probit Procedure" [6] which determines the best cumulative distribution function based on the assumption the underlying population has a normal distribution. The results of the analysis of Reference 5 are shown in Figure 3. (The independent variable in Figure 3, the Thoracic Trauma Index, is dependent on: (1) age, (2) upper rib acceleration, (3) 12th thoracic vertebra acceleration, and (4) mass. The development of this parameter is

discussed in Reference 5. For the purpose of the current paper, look at the TTI as an independent parameter and avoid asking what it is.) Figure 3 plots the three Probit solution curves where the all-or-nothing response is defined respectively as AISIHT ≥ 3 , ≥ 4 , and ≥ 5 . (The injury analysis efforts related thoracic response to the resulting injuries. The AIS was for all lesions occurring to anatomical structures on or within the thoracic cage. This concept includes such organs as the liver, kidney, and spleen, which by the conventional AIS definition are abdominal organs. To distinguish this classification from the conventional AIS rating scheme, the acronym AISHT has been assigned. AISIHT represents the level of highest hard thoracic AIS which was observed.) As can be seen, slight inconsistencies exist at the high end of the AISIHT ≥ 3 and AISIHT ≥ 4 curves as well as at the low end of the AISIHT ≥ 4 and AISIHT ≥ 5 curves in Figure 3. Also, the Probit Procedure used for Figure 3 assumes that the underlying injury population is normal with respect to the independent variable, TTI, but no evidence exists to take this as a given. How well does the Maximum Likelihood approach with the Weibull Distribution Function treat the same data set? To simplify the analysis, the AISIHT curves ≥ 3 , ≥ 4 , and ≥ 5 will be discussed separately and in sequence.

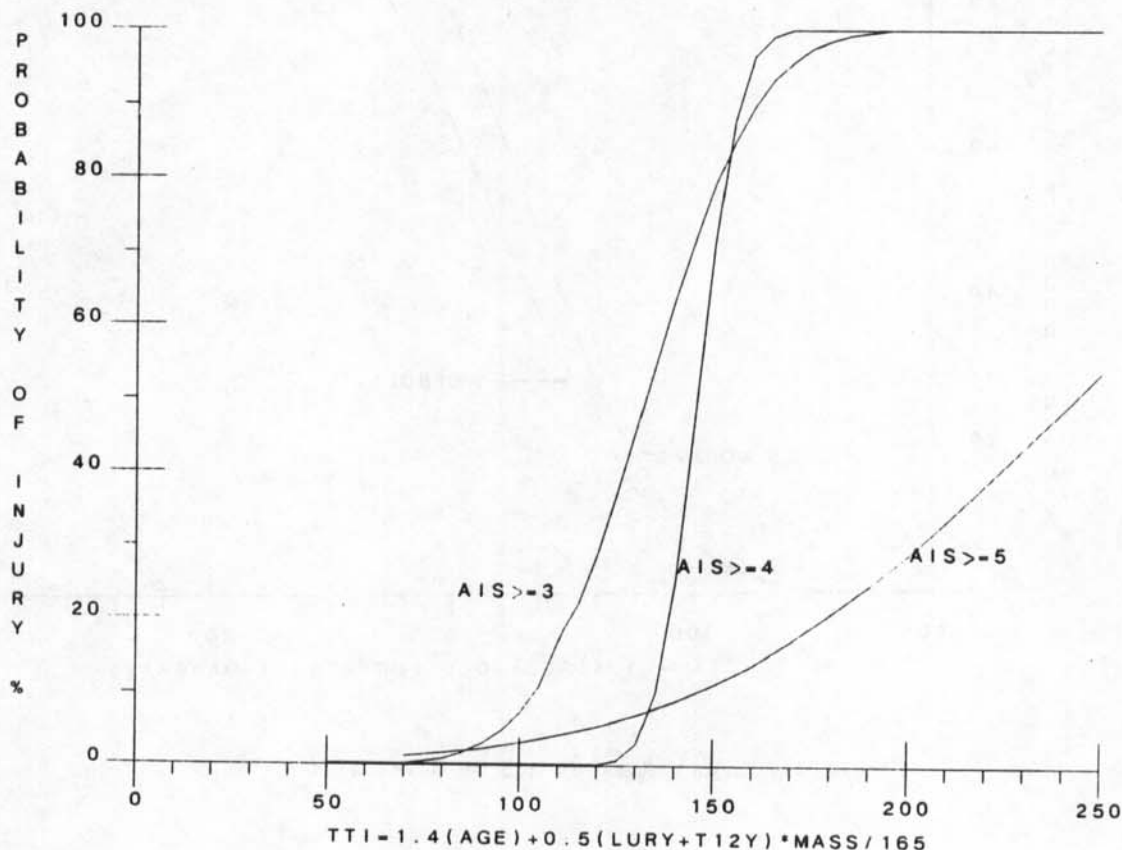


Figure 3 - Injury Risk vs. TTI from Reference 5.

The old normal analysis (Probit Analysis) and the Weibull approach are compared in Figure 4. Both methods give roughly the same probability curve. One question which could be asked is, "Which method gives the greater maximum likelihood function?" The normal approach gives an L value of $4.9 * 10^{-9}$ and the Weibull an L of $2.0 * 10^{-8}$; so the Weibull approach does a better job. Another question to be asked is, "How do the probability density functions of the two methods compare?"

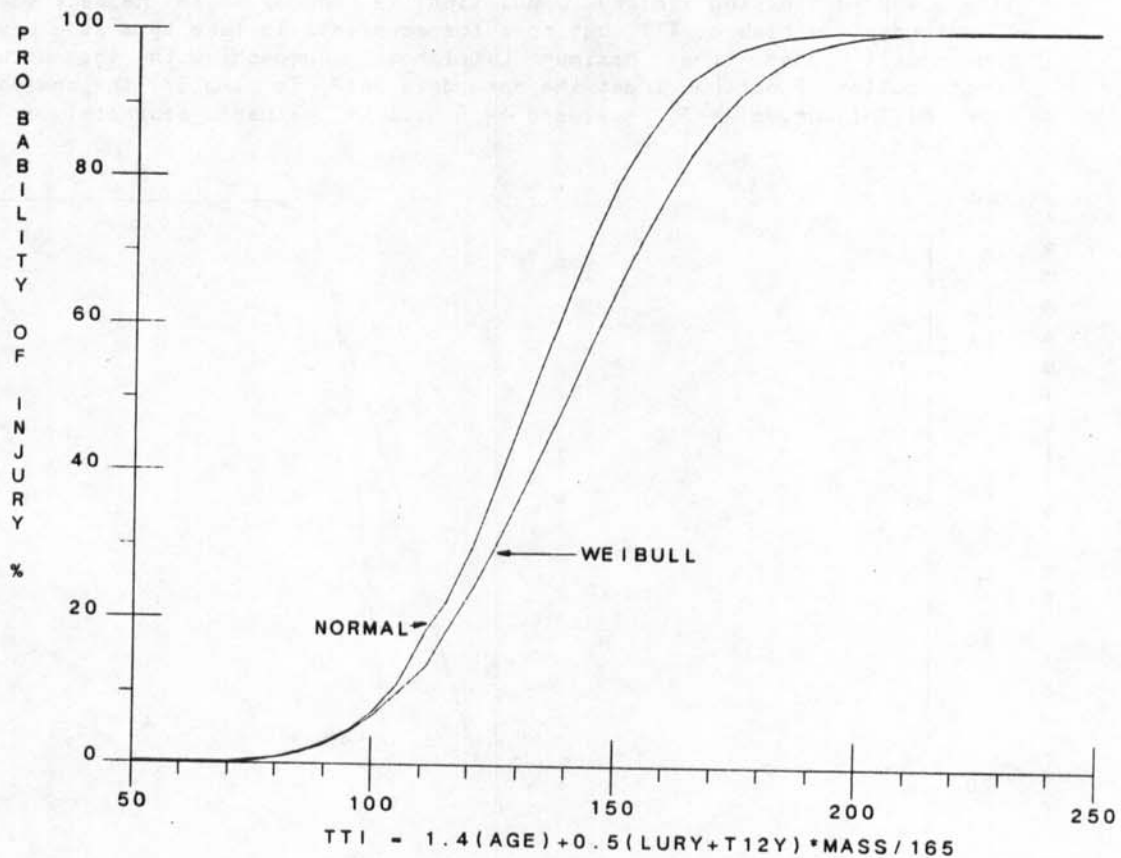


Figure 4 - Injury Risk of AIS ≥ 3 vs. TTI.

Recall from the Finney excerpt that there is a function $F(Z)$ which gives the percentage responding against an exposure by a steadily rising curve having a sigmodal shape as shown in Figure 5. Big $F(Z)$ is called the cumulative distribution function by statisticians. Big $F(Z)$ was arrived at by integrating small $f(Z)$ which is called the probability density function by statisticians. Small $f(Z)$ as shown in Figure 6 should be interpreted such that the area between any two ordinates represents the proportion of subjects having tolerances between these limits.

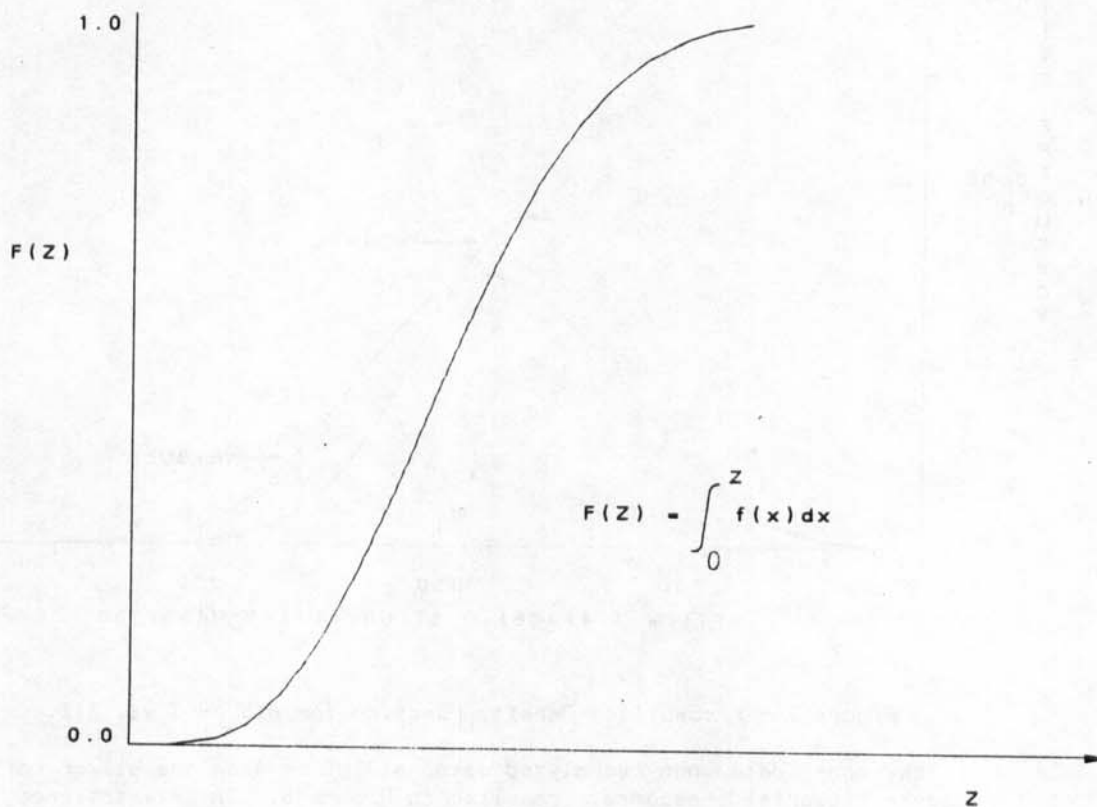


Figure 5 - Function $F(Z)$ Gives Percentage Responding Against an Exposure Z .

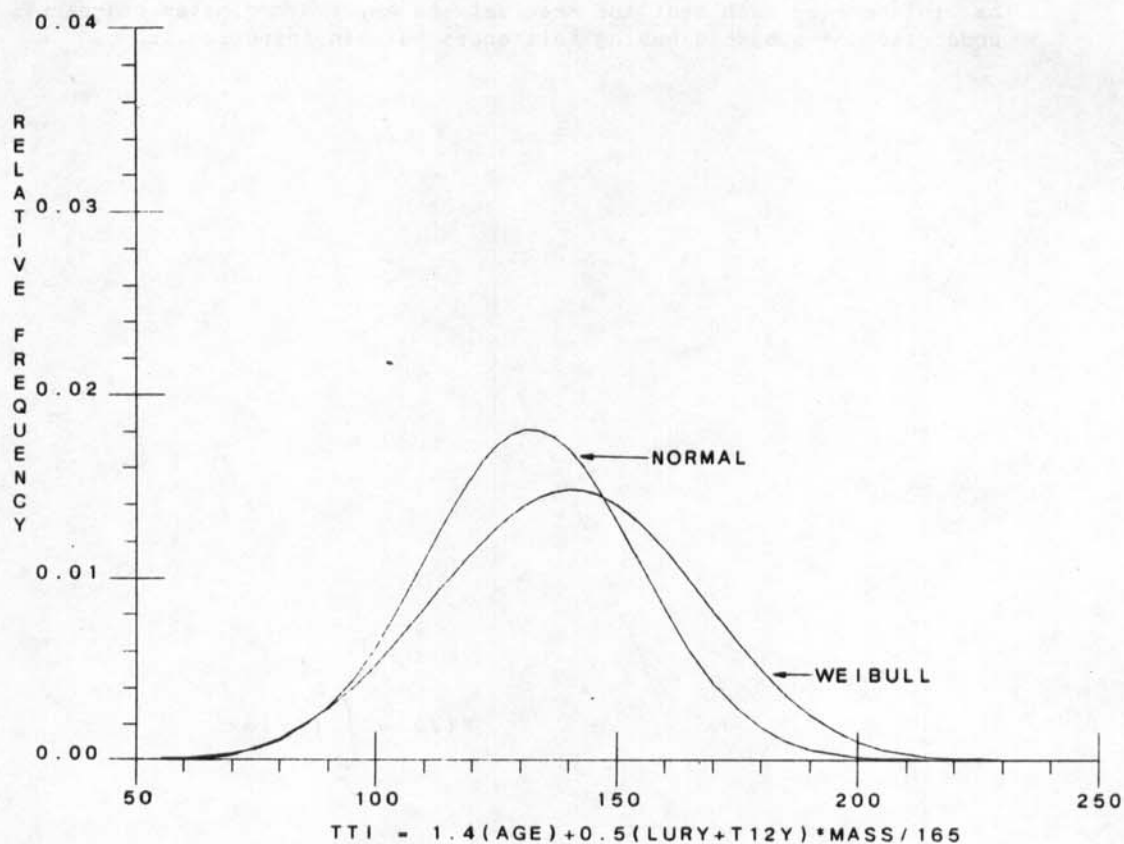


Figure 7 - Probability Density Function for AIS ≥ 3 vs. TTI.

The same data when reanalyzed using AISIHT ≥ 4 as the all-or-nothing response (quantal response) results in Figure 8. In this instance, the probability curves (cumulative distribution functions) for the Probit and Weibull approach are beginning to separate slightly. The maximum likelihood for the Probit is 3.1×10^{-7} and for the Weibull is 4.3×10^{-5} . The probability density functions for the Probit and Weibull are shown in Figure 9. Both look normal but the underlying populations are starting to diverge.

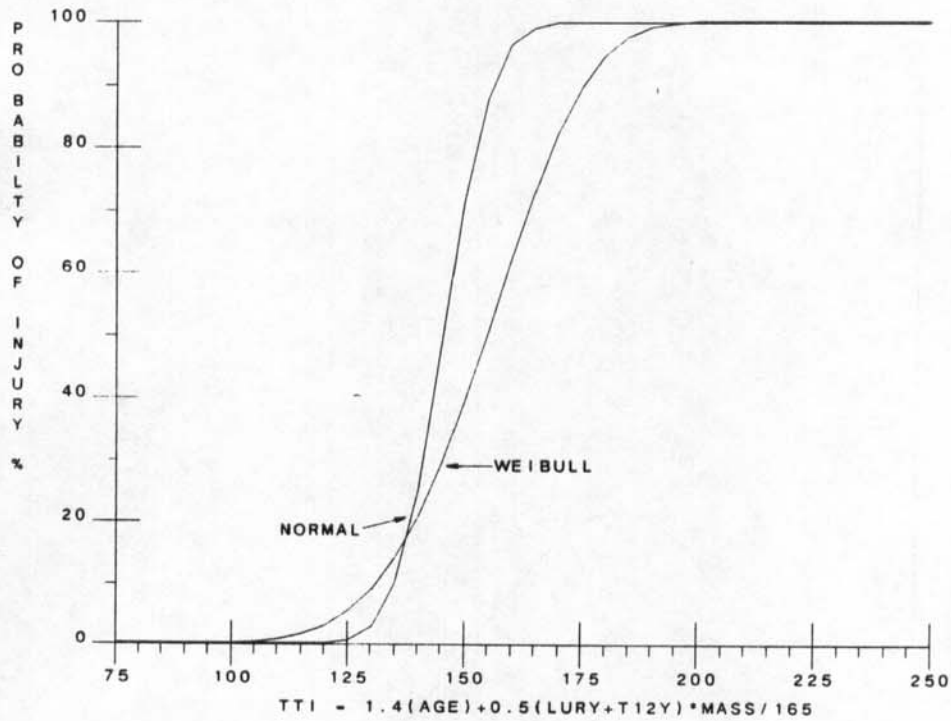


Figure 8 - Injury Risk of AIS ≥ 4 vs. TTI.

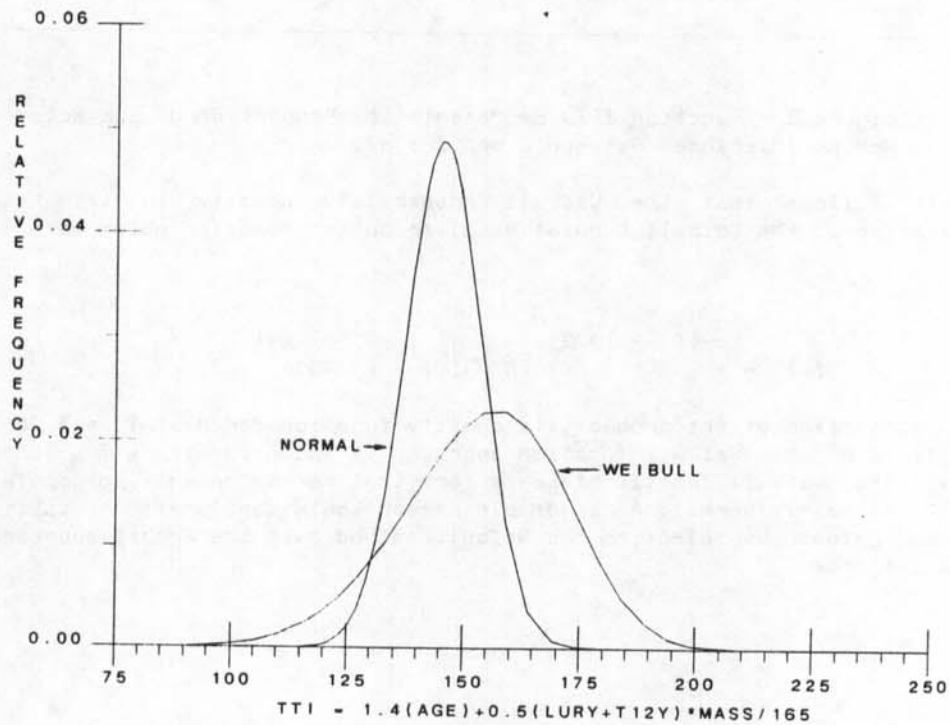


Figure 9 - Probability Density Function for AIS ≥ 4 vs. TTI.

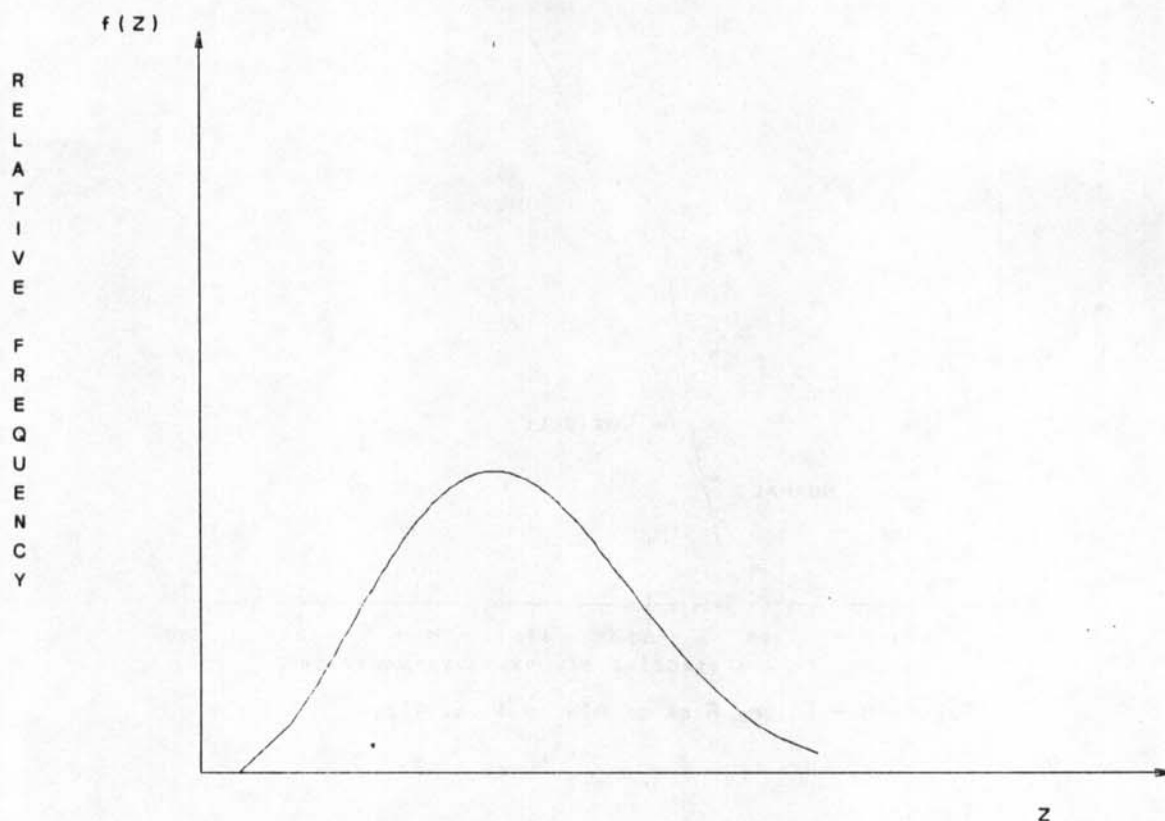


Figure 6 - Function $f(Z)$ Represents the Proportion of Subjects Having Tolerances Between Z and $Z + dZ$.

It follows that the Weibull probability density function is the derivative of the Weibull cumulative distribution function which is:

$$f(Z) = e^{-[(Z - \gamma)/\alpha]^\beta} [\beta/\alpha][(Z - \gamma)/\alpha]^{\beta-1}$$

The comparison of the probability density function for AISHT ≥ 3 for the Probit and the Weibull function approach is shown in Figure 7. For this case, the Weibull function takes a form that has an underlying population which is nearly normal. A reasonable person would conclude there is little to be gained by selecting the Weibull method over the Probit approach in this instance.

Finally, the side impact data was analyzed using AISIHT ≥ 5 as the criteria for the quantal response. The cumulative distribution functions are shown in Figure 10; the probability curves are definitely different. The maximum likelihood for the Probit is 1.7×10^{-7} and for the Weibull is 2.7×10^{-7} . The juxtaposition of the two methods in the probability density function versus independent variable format of Figure 11 shows that the Weibull underlying population is skewed to the right.

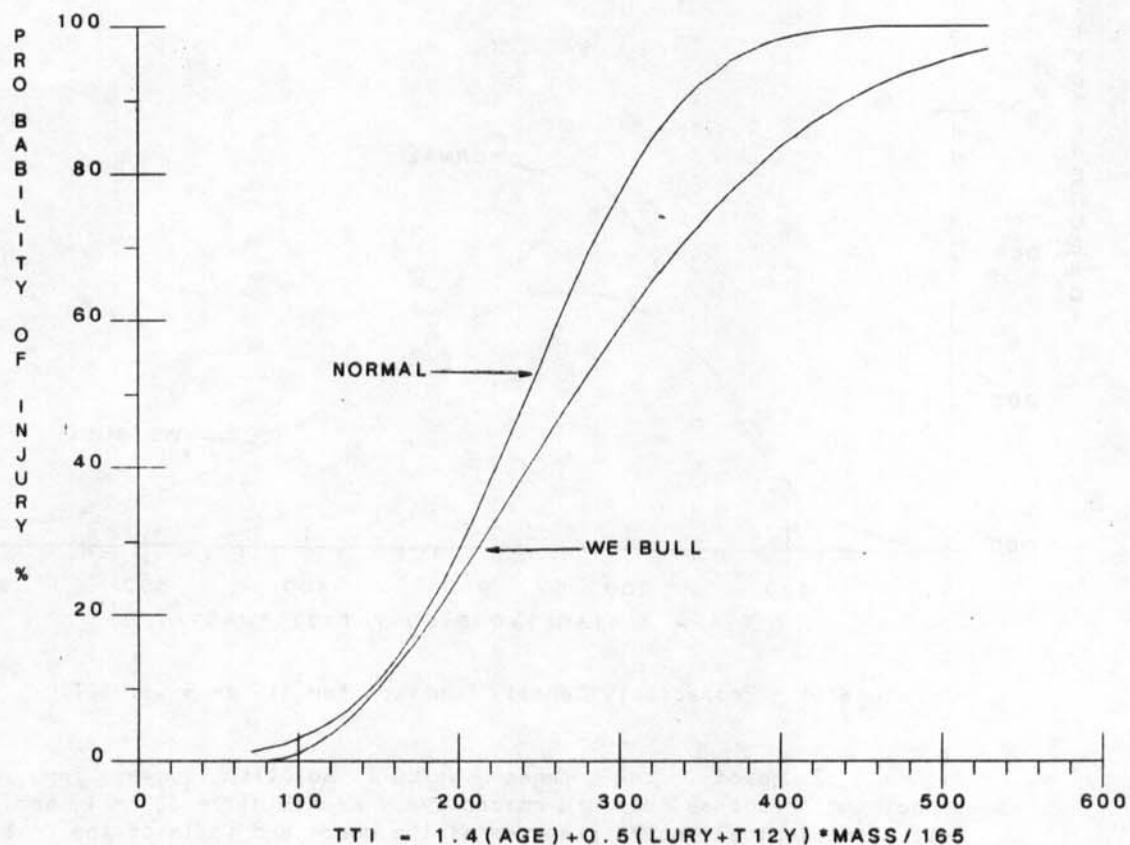


Figure 10 - Injury Risk of AIS ≥ 5 vs. TTI.

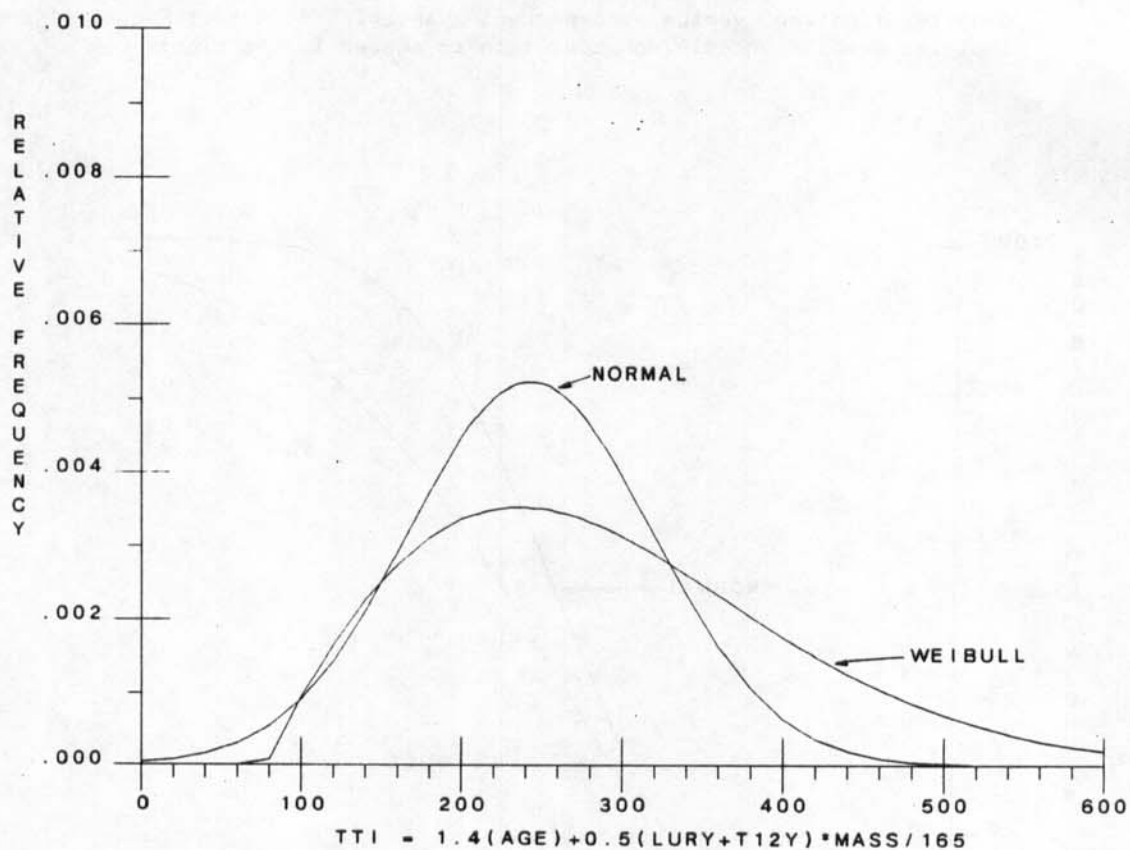


Figure 11 - Probability Density Function for AIS ≥ 5 vs. TTI.

Figure 12 plots the three Weibull solution curves for the all-or-nothing response defined respectively as AISIHT ≥ 3 , ≥ 4 , and ≥ 5 . Note that logical inconsistencies at the heads and tails of the Probit method solutions are hardly present in the Weibull method solutions. For the examples presented in this section of the paper, the maximum likelihood for the Weibull approach is slightly greater to greater than that for the Probit approach. Finally, unsubstantiated assumption of an underlying normal population is not required by the Weibull approach. For this side impact cadaver data set, the Weibull approach is superior.

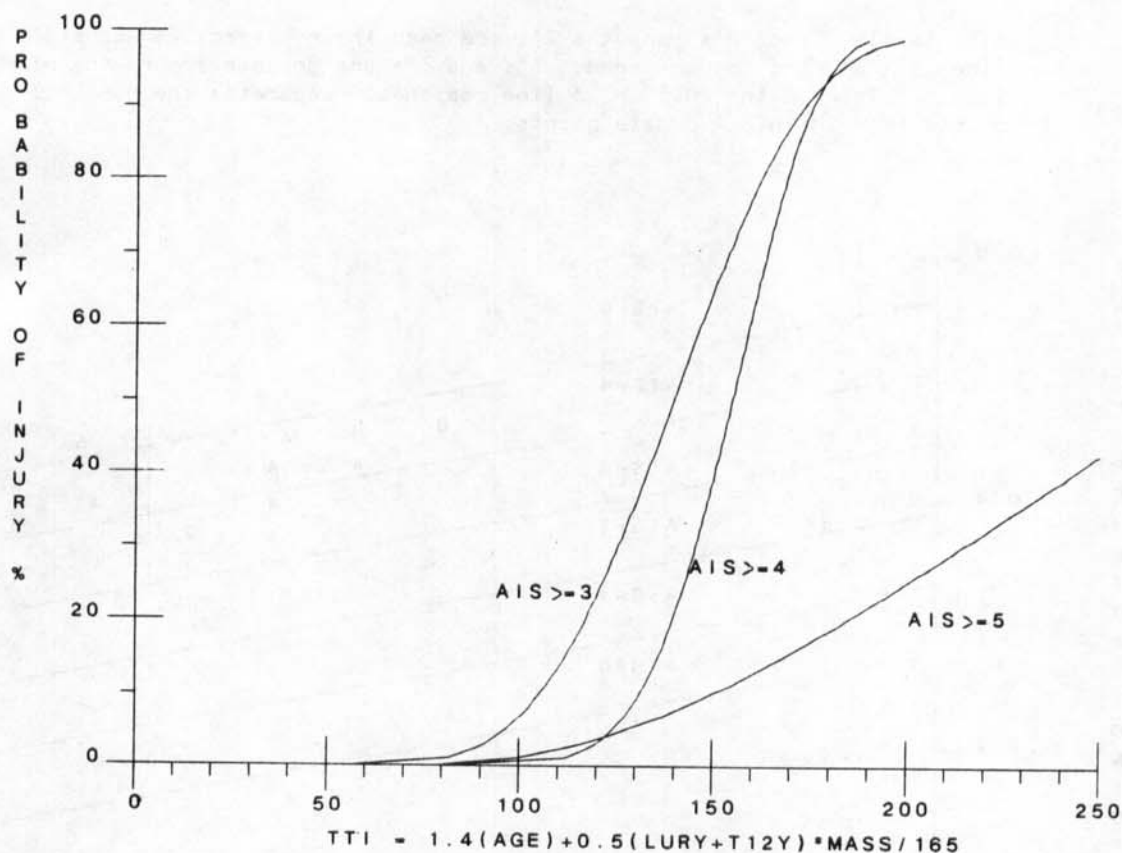


Figure 12 - Injury Risk vs. TTI, Weibull Solution.

METHOD APPLIED TO FRONTAL PENDULUM DATA

Another source of cadaver data are the blunt frontal pendulum impacts frequently referred to as the Kroell Data and sponsored by General Motors [7-9]. A subset of this data -- 24 tests -- was analyzed by Neathery et al. using regression techniques [10]. The final equation developed was

$$AIS = 17.4338 P/D + 0.03128 \text{ Age} - 5.1508$$

where P is chest penetration,

D is chest depth,

Age is in years,

and the multiple correlation coefficient, R, is 0.87.

The constant AIS lines (derived by setting the AIS variable in the above equation from Reference 10 equal to the particular AIS number to be the constant) are overlayed on the cadaver data points in Figure 13. Three things are evident. First, at the higher AIS levels, the AIS = 4, 5, and 6 data points appear to group together as the constant AIS lines move away -- in a linear fashion -- from the grouping. In other words the equation starts to lose its predictive capability as P/D increases beyond about .42 as was first pointed out by Viano [11]. Second, at the lower

AIS levels, the 0's and the 2's are near their respective constant AIS lines. On the other hand, 1's and 3's are grouped around the AIS = 2 line. Third, the AIS = 3 line reasonably separates the AIS ≤ 3 data points from the AIS > 3 data points.

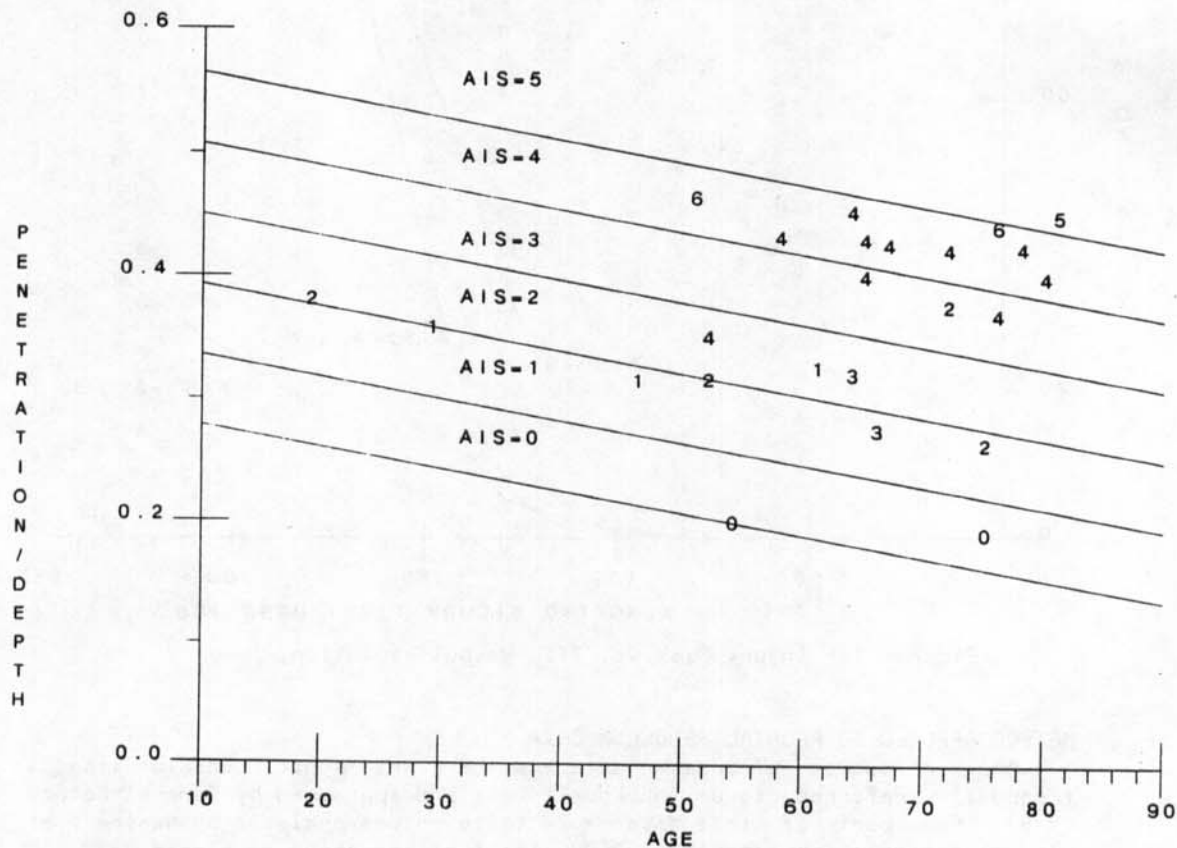


Figure 13 - Constant AIS Lines on Data from Reference 10.

The same data when analyzed by the Weibull approach -- where the criteria for response and non-response is AIS > 3 and AIS ≤ 3 respectively -- results in the probability curve of Figure 14. The Weibull approach shows that the tolerance to AIS > 3 goes through a transition about the region where the equation of Reference 10 predicts the AIS is equal to 3. Basically, the Weibull approach shows the salient behavior of the cadaver data without running the risk of inaccurately predicting at the lower and higher values of the AIS.

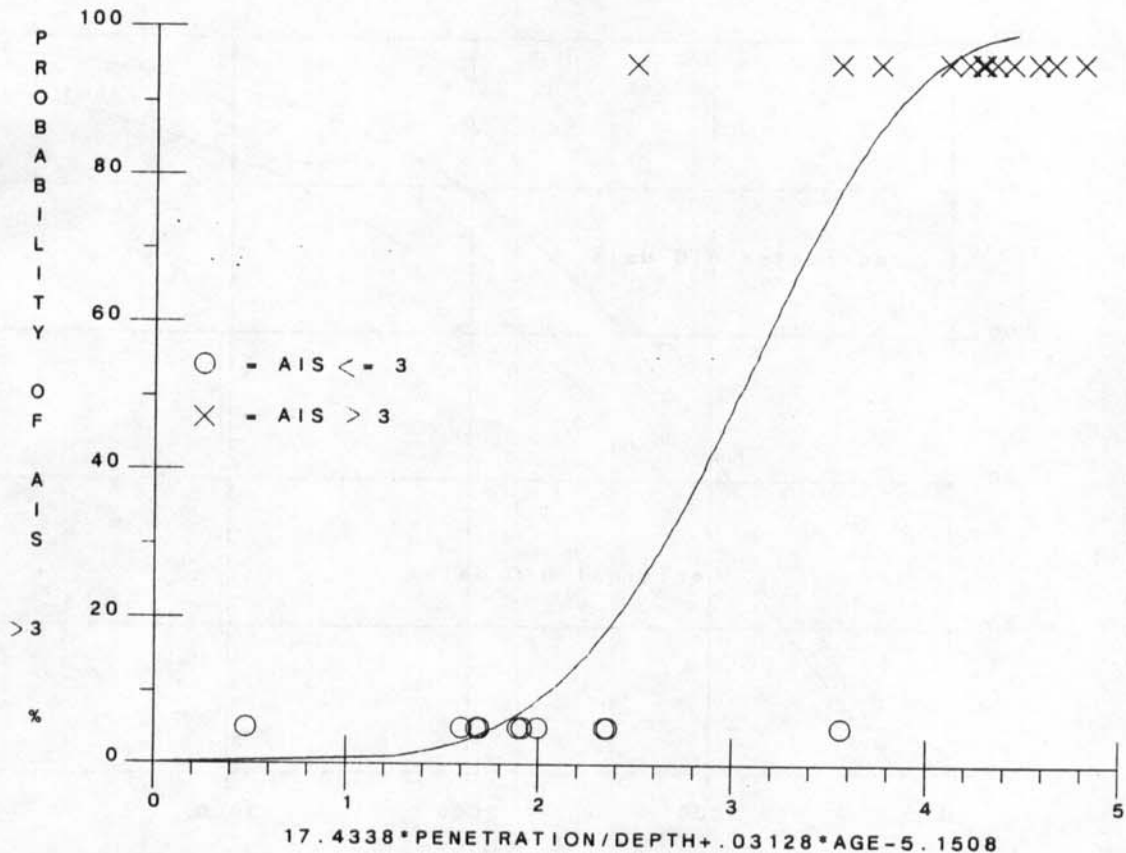


Figure 14 - Injury Risk vs. Neathery's Equation.

METHOD APPLIED TO HEAD INJURY CRITERIA DATA

In the Volvo paper, Reference 1, is an example using the Weibull Function/Maximum Likelihood approach on 54 cadaver head impact tests. The original cranial fracture data is taken from Reference 12. Using our in-house computer program, a solution identical to Volvo's was found. That Weibull Cumulative Distribution Function is found in Figure 15 with the label "Original HIC Data." Subsequently, Dr. Priya Prasad of Ford Motor Company found in the original literature that six of the tests labeled not skull fractures were in fact fractures. The Weibull simulation was rerun with the corrections and a new solution was found which is labelled "Corrected HIC Data" in Figure 15. The 50th percentile probability region is found at about a 1500 HIC level with the original data but is shifted to around a 1000 HIC with the corrected data. The point of this exercise is that the Weibull Function/Maximum Likelihood approach lends itself readily to sensitivity studies.

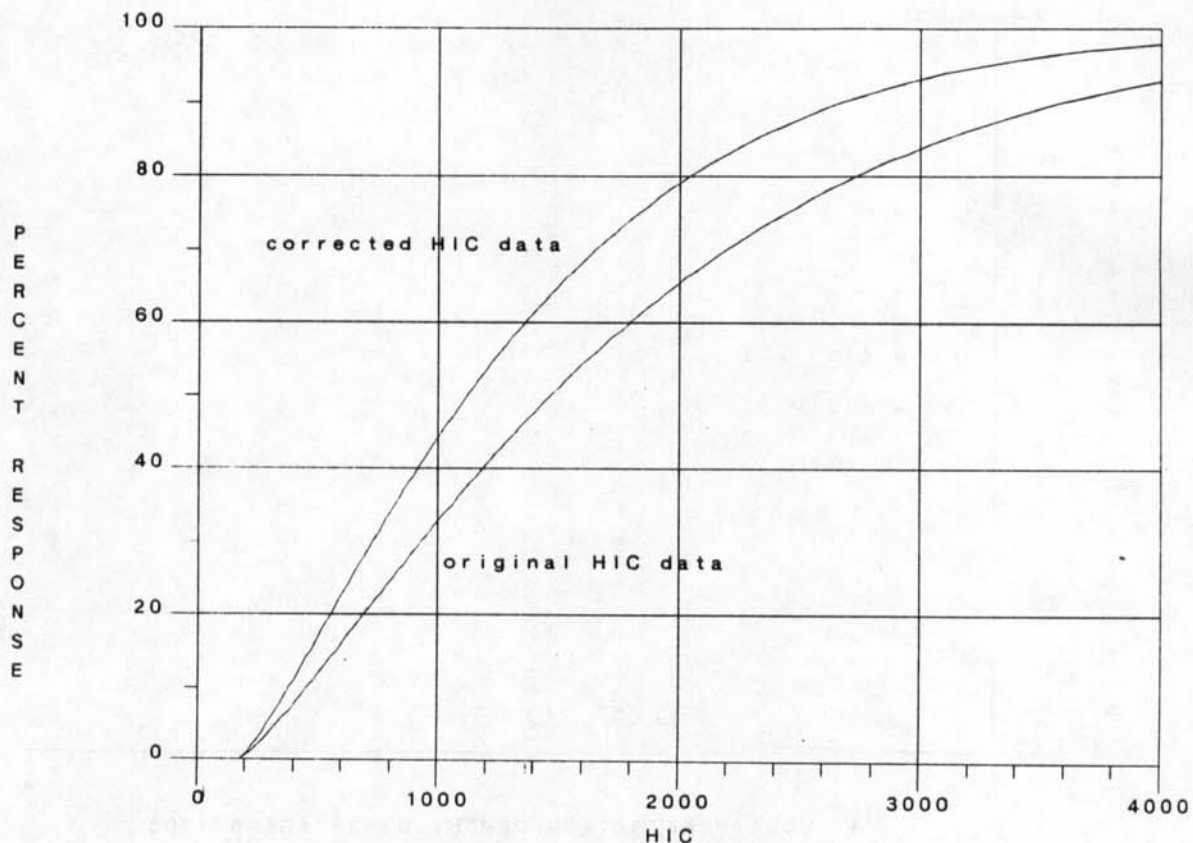


Figure 15 - Fracture Risk vs. HIC data from Reference 12.

CONCLUSIONS

The first conclusion is that it is a relatively simple task to implement the Weibull Cumulative Distribution Function/Maximum Likelihood approach given the availability of commercial support software and small, fast computers with large memory.

The second conclusion pertains to the comparison of the Probit Method with the Weibull Method for the analysis of a side impact cadaver data set. The Probit analysis leads to probability curves which have inconsistent high and low values when considering different levels of AIS values. The Weibull approach leads to probability curves which have a smoother transition at the high and low ends.

The third conclusion is that if the Maximum Likelihood is taken as the criteria of merit, then the Weibull approach has a higher Maximum Likelihood value than the Probit approach for our examples.

The fourth conclusion is that the Weibull approach readily allows for variation of the independent variables to determine the impact on the probability curve(s).

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